In solids a single shock front is frequently unstable and a compressive wave propagates as two or more shock fronts. The stability criterion is derived by assuming the shock to consist of two fronts and comparing their relative speeds.[2]

Assume that the first shock, travelling with velocity U_B in laboratory coordinates, transforms the state to $P_B - P_A$, $u_B - u_A$. Its velocity with respect to the material behind it is then $U_B - u_B$. The velocity of the second shock is U_C in laboratory coordinates, and $U_C - u_B$ with respect to the material ahead of it. The final state is P_C, V_C.

Employing the jump conditions, Eqs. (1) and (2), these velocities can be written

$$U_B - u_B = V_B \sqrt{(P_B-P_A)/(V_A-V_B)}$$

$$U_C - u_B = V_B \sqrt{(P_C-P_B)/(V_B-V_C)}$$

If the assumed second shock travels faster than the first a single front is stable. Thus, the condition for stability is:

$$(P_C-P_B)/(V_B-V_C) > (P_B-P_A)/(V_A-V_B)^*$$

Graphically this means that the Rayleigh line joining point P_B, V_B with P_C, V_C is steeper (more negative) than that joining P_B, V_B with P_A, V_A. (Figure 2.)

A cusp in the P-V curve as indicated at point B in Figure 2 occurs in the majority of solids at the elastic yield point. Because the strain is one-dimensional, shear stresses are developed by a plane wave. In an elastic solid the relation is:[11]

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*Although this derivation is not entirely rigorous, more detailed examination shows the result to be correct.

$$\tau = [(1-2\nu)/(2(1-\nu))]P$$

where τ is the shear stress and ν is Poisson's ratio. Clearly, as P increases so does τ until the material yields. The elastic portion of the P-V curve has slope,

$$-(dP/dV) = K + (4/3)\mu,$$

where K is the bulk modulus and μ the shear modulus. In the plastic region above the yield point the slope is reduced to (neglecting work hardening):

$$-(dP/dV) = K$$

Accordingly there is a range of shock amplitudes for which the stability criterion is not satisfied. Since K increases with pressure, however, the unstable region is bounded at higher pressures (Point D in Fig. 2) as well as at the elastic limit.

Instability can also result from phase transformations. The isothermal volume discontinuity of a first-order phase change frequently corresponds to a cusp in the R-H curve and the consequent separation of shock fronts is very suitable for experimentally detecting the transition and measuring its pressure.

3. Reflections at Interfaces

For plane waves the interaction with a boundary of different shock impedance is characterized by continuity of the stress normal to the boundary and the mass velocity. For this reason it is convenient to consider the relations between stress and particle velocity obtaining in shock transitions and in rarefactions.

The shock velocity can be eliminated from Eqs. (1) and (2) to give